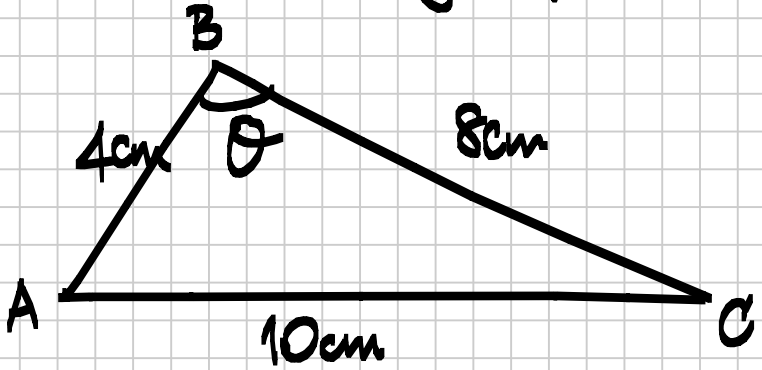
C2 Exercise 2E (cosine rule to find the angle)

Note Title 05/02/2007

1a In each triangle find θ



$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$100 = 64 + 16 - 2(8)(4)\cos B$$

$$\Rightarrow \quad 64\cos B = 80 - 100$$

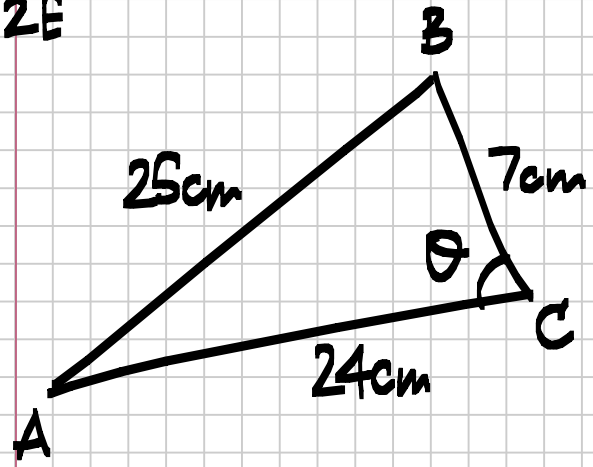
$$\Rightarrow \quad \cos B = \frac{-20}{64} \quad \Rightarrow B = \cos^{-1}\left(\frac{-5}{16}\right)$$

$$B \simeq 108.2^\circ$$

$$\theta \simeq 108^\circ$$

C2 Ex2E

1b



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$25^2 = 7^2 + 24^2 - 2(7)(24)\cos C$$

$$336\cos C = 49 + 576 - 625$$

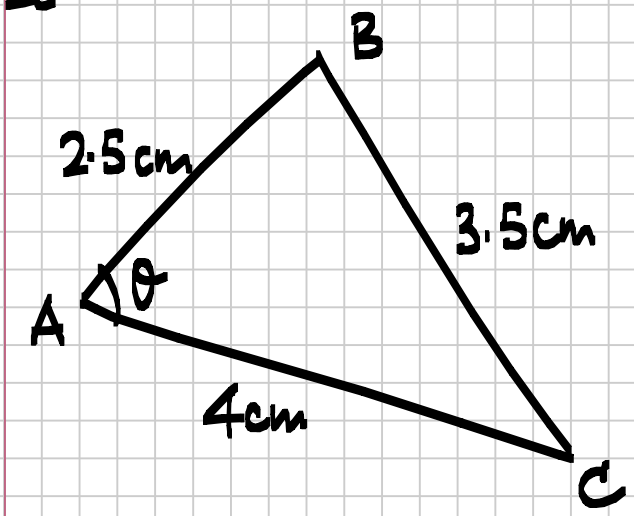
$$\cos C = 0$$

$$\Rightarrow C = 90^\circ$$

$$\theta = 90^\circ$$

C2 Ex 2E

1c



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$(3.5)^2 = 4^2 + (2.5)^2 - 2(4)(2.5)\cos A$$

$$\Rightarrow \quad 20\cos A = 16 + 6.25 - 12.25$$

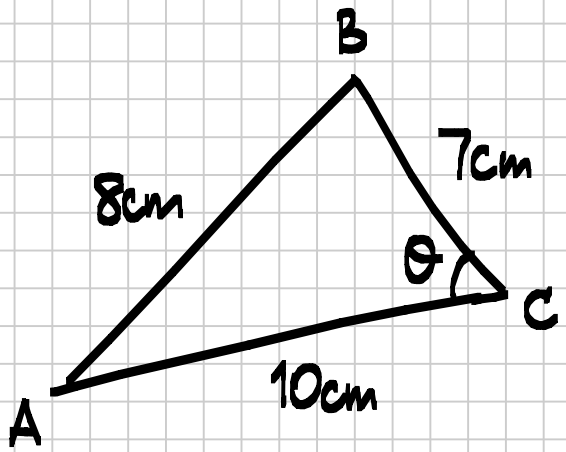
$$\cos A = \frac{10}{20}$$

$$A = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

$$\theta = 60^\circ$$

C2 Ex 2E

1d



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\Rightarrow \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \cos C = \frac{49 + 100 - 64}{140}$$

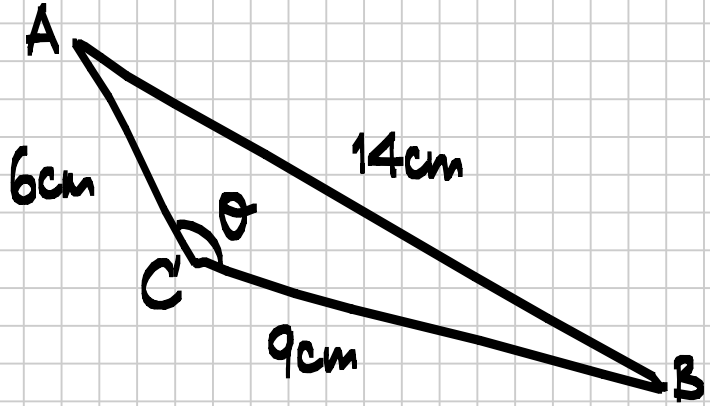
$$\cos C = \frac{85}{140}$$

$$C \simeq 52.62°$$

$$\theta \simeq 52.6°$$

C2 Ex 2E

1e



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\Rightarrow \cos\theta = \frac{a^2 + b^2 - c^2}{2ab}$$

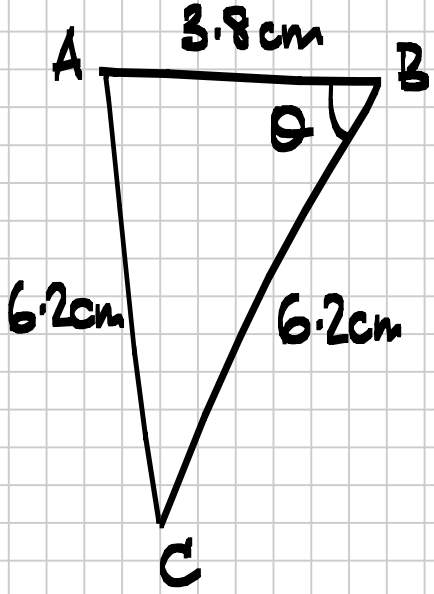
$$\Rightarrow \cos\theta = \frac{81 + 36 - 196}{2(9)(6)}$$

$$\cos\theta = \frac{79}{108}$$

$$\theta \simeq 43.0^\circ$$

C2 Ex 2E

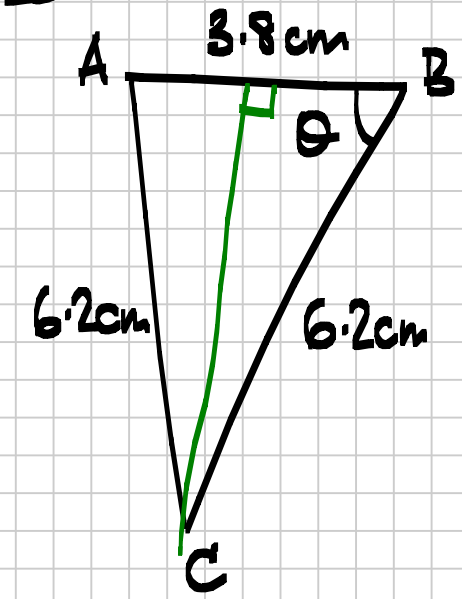
1f



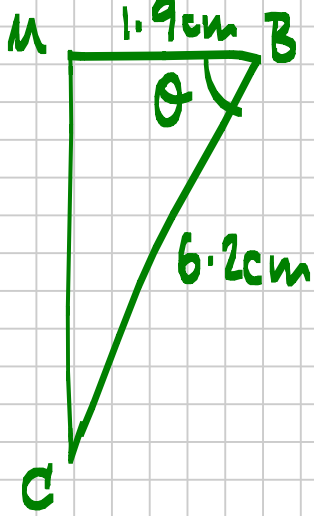
$$b^2 = a^2 + c^2 - 2ac\cos B$$

C2 Ex 2E

1f



Hang on, $\triangle ABC$ is isosceles, so why not cut it in half and use trigonometry on this bit:

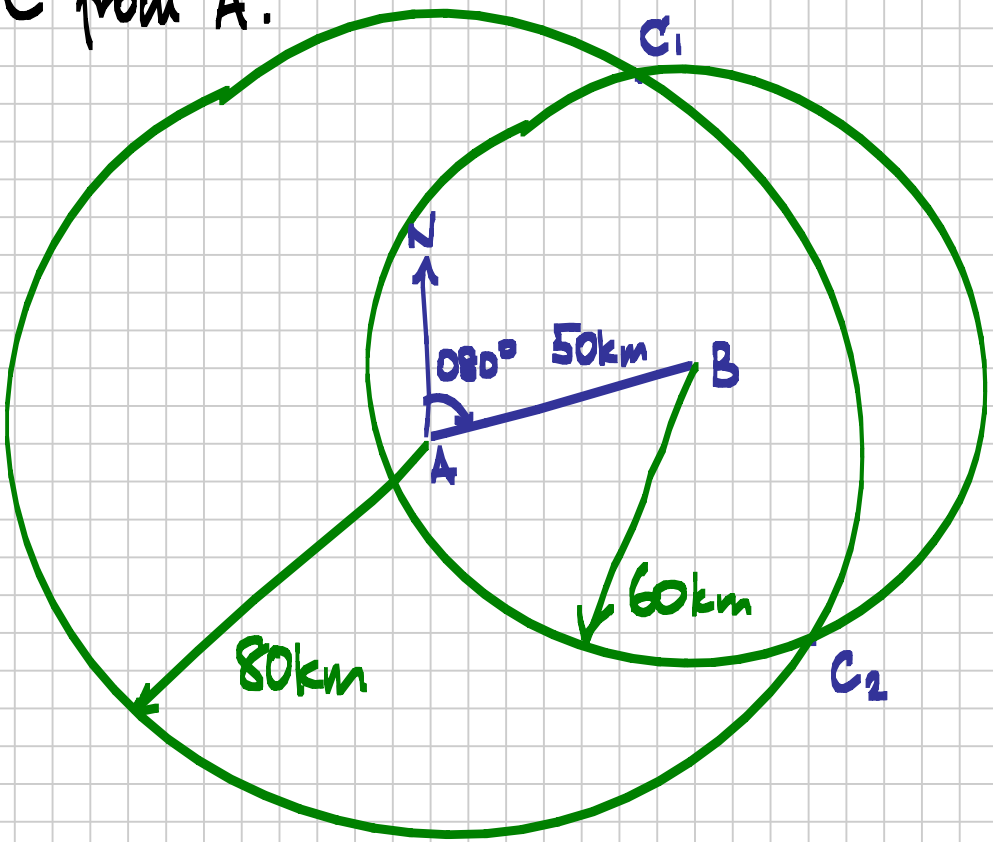


$$\cos\theta = \frac{1.9}{6.2}$$

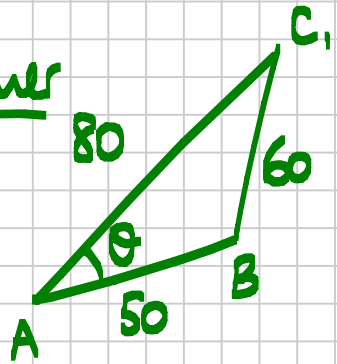
$$\Rightarrow \theta = \cos^{-1}\left(\frac{1.9}{6.2}\right)$$

C2 Ex 2E

2 A helicopter flies on a bearing of 080° from A to B, where AB = 50 km. It then flies for 60km to a point C. Given that C is 80km from A, calculate the bearing of C from A.



either



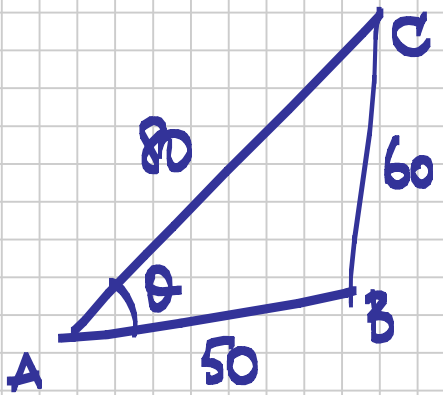
the bearing of C_1 from A is $080 - \theta$.

or

the bearing of C_2 from A is $080 + \theta$.

C2 Ex 2E

2



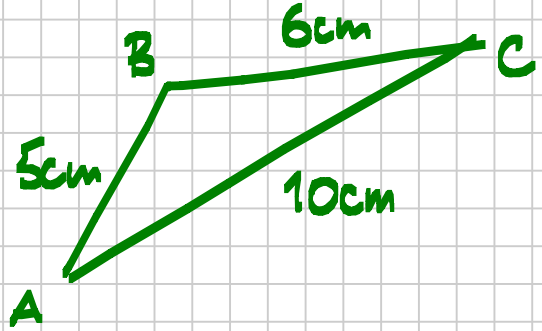
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \cos\theta = \frac{80^2 + 50^2 - 60^2}{2 \times 50 \times 80}$$

C2 Ex 2E

3 In $\triangle ABC$, $AB = 5cm$, $BC = 6cm$ and $AC = 10cm$.
Calculate the value of the smallest angle.

The smallest angle must be opposite the smallest side: here it's angle C:



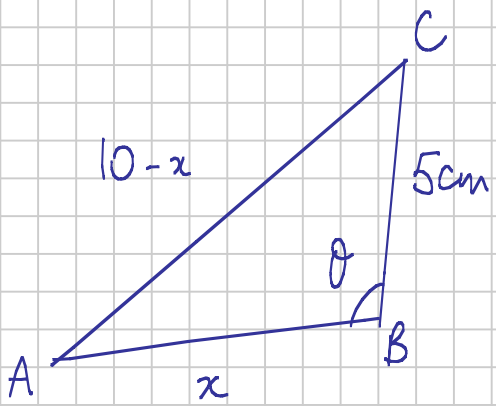
C2 Ex 2E

4 In ΔABC, $AB = 9.3\text{cm}$, $BC = 6.2\text{cm}$ and $AC = 12.7\text{cm}$

5

6 In triangle ABC, $AB = x$ cm $BC = 5$cm and $AC = (10-x)$

(a) Show that $\cos A\hat{B}C = \dfrac{4x-15}{2x}$.



put $A\hat{B}C = \theta$

Then $\cos\theta = \dfrac{a^2 + c^2 - b^2}{2ac}$

$$\cos\theta = \frac{5^2 + x^2 - (10-x)^2}{2(5)(x)}$$

$$\cos\theta = \frac{25 + \cancel{x^2} - 100 + 20x - \cancel{x^2}}{10x}$$

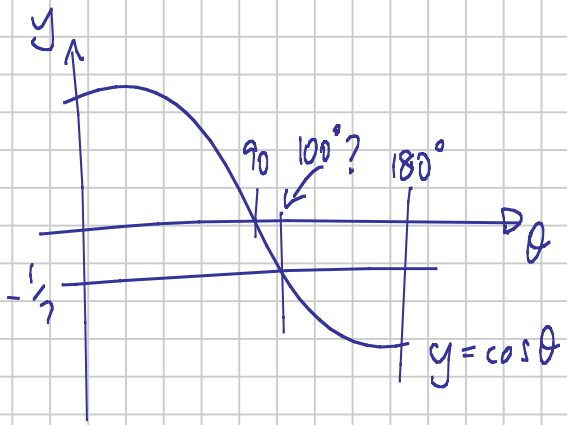
6(a)

$$\Rightarrow \cos\theta = \frac{20x - 75}{5(2x)}$$

$$\Rightarrow \cos\theta = \frac{4x - 15}{2x}$$

$$\Rightarrow \cos A\hat{B}C = \frac{4x - 15}{2x} \quad \square.$$

6b Given that $\cos A\hat{B}C = -\frac{1}{7}$, work out the value of x



$$\frac{4x - 15}{2x} = -\frac{1}{7}$$

$$7(4x - 15) = -1(2x)$$

$$28x - 105 = -2x$$

$$30x = 105$$

$$6x = 21$$

$$2x = 7$$

$$x = 7/2 \text{ or } 3\frac{1}{2} \text{ or } 3.5$$